

Lorentz Invariance Violation in Modified Gravity

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We consider an environmentally dependent violation of Lorentz invariance in scalar-tensor models of modified gravity where General Relativity is retrieved locally thanks to a screening mechanism. We find that fermions have a modified dispersion relation and would go faster than light in an anisotropic and space-dependent way along the scalar field lines of force. We comment briefly on the OPERA claims and argue that this type of Lorentz violation might be due to chameleons. We suggest that measuring the speed of neutrinos emitted radially from the surface of the earth, at different energies, and observed on the other side of the earth would provide a test of the Lorentz invariance violation induced by modified gravity effects.

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The discovery of the acceleration of the Universe has led to a flurry of scenarios involving scalar fields and leading to different types of modified gravity models [1]. All of them allow for large deviations from General Relativity on astrophysical scales while preserving Newton's law locally in the solar system and in laboratories on earth. This is achieved thanks to screening features such as the Vainshtein mechanism for theories with higher order derivative self interactions[2], or the chameleon[3], symmetron[4] or Damour-Polyakov properties[5] for theories with non-linear effective potentials in the presence of pressure-less matter. Recently and after the claim of super-luminal propagation of neutrinos by the OPERA experiment[6], it has been suggested that fermions may travel faster than the speed of light in dense environments where the presence of matter offers a breaking of Lorentz invariance[7]. This was further pursued in [8, 9] and then in [10] where Galileons were used to describe the OPERA claims although failing to respect the tight bounds on the deviation of the electron speed from the speed of light. In this work, we will describe a general mechanism which leads to anomalously fast fermions and where the breaking of Lorentz invariance by a two-tensor appears naturally in models of modified gravity with screening properties. Chameleon, symmetron and Galileon models can reproduce phenomena of the type indicated by the preliminary OPERA publication. Only models with a thin shell screening mechanism, i.e. chameleons, are compatible with the constraints on the speed of charged leptons.

We consider the action governing the dynamics of a scalar field ϕ in a scalar-tensor theory of the general form

$$S = \int d^4x \sqrt{-g} \left\{ \frac{m_{\text{Pl}}^2}{2} R - F(\partial_\mu \phi, \partial_\mu \partial_\nu \phi) - V(\phi) \right\} + \sum_i S_m^i(\psi_m^{(i,j)}, \tilde{g}_{\mu\nu}^i), \quad (1)$$

where

$$S_m^i(\psi_m^{(i,j)}, \tilde{g}_{\mu\nu}^i) = \int d^4x \sqrt{-\tilde{g}_i} \mathcal{L}_m^i(\psi_m^{(i,j)}, \tilde{g}_{\mu\nu}^i) \quad (2)$$

is the action in the i th sector of the model, g is the determinant of the metric $g_{\mu\nu}$, R is the Ricci scalar and $\psi_m^{(i,j)}$ are various matter fields labeled by j interacting with the metric $\tilde{g}_{\mu\nu}^i$ in the Lagrangian \mathcal{L}_m^i . When $F(\partial_\mu \phi, \partial_\mu \partial_\nu \phi) = \frac{1}{2}(\partial\phi)^2$, the field is canonically normalised. More complex functions $F(\partial_\mu \phi, \partial_\mu \partial_\nu \phi)$ appear in the Galileon scenario for instance [11]. A key ingredient of the model is the coupling of ϕ with matter particles. More precisely, the excitations of each matter field $\psi_m^{(i,j)}$ couple to a metric $\tilde{g}_{\mu\nu}^i$ which is related to the Einstein-frame metric $g_{\mu\nu}$ by

$$\tilde{g}_{\mu\nu}^i = A^2(\phi) g_{\mu\nu}^i \quad (3)$$

where

$$g_{\mu\nu}^i = g_{\mu\nu} + \frac{2\partial_\mu \phi \partial_\nu \phi}{M_i^4} \quad (4)$$

depends on each sector of the theory. For instance, the fermion kinetic terms may couple to a different metric from the gauge kinetic terms.

In the following, we will assume that the bi-metric term $\frac{\partial_\mu \phi \partial_\nu \phi}{M_i^4}$ is a small correction to $g_{\mu\nu}$. In the context of extra dimensional models where matter lives on a brane, the scalar field can be seen as parameterising the normal to the brane, the coupling function $A(\phi)$ arises from the warping of the bulk metric while the bilinear term reflects the coupling of matter to the induced metric on the brane. Defining the energy momentum tensor in each sector as $T_i^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_m^i}{\delta g_{\mu\nu}}$ and expanding the action to linear order, we find that the scalar field couples derivatively to matter

$$S = \int d^4x \sqrt{-g} \left\{ \frac{m_{\text{Pl}}^2}{2} R - F(\partial_\mu \phi, \partial_\mu \partial_\nu \phi) - V(\phi) \right\} + \sum_i \int d^4x \sqrt{-g} \frac{\partial_\mu \phi \partial_\nu \phi}{M_i^4} T_i^{\mu\nu} + \sum_i \int d^4x \sqrt{-g} A^4(\phi) \mathcal{L}_m(\psi_m^{(i,j)}, A^2(\phi) g_{\mu\nu}), \quad (5)$$

As soon as $\theta_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi$ does not vanish due to the presence of matter, we find that Lorentz invariance is broken and a new Lorentz invariance violating coupling to the matter energy momentum tensors is present in the model[9].

Massless fermions with the action $S_F = -\int d^4x \sqrt{-g} \frac{i}{2} (\bar{\psi} \gamma^\mu D_\mu \psi - (D_\mu \bar{\psi}) \gamma^\mu \psi)$ have the energy momentum tensor $T_{\mu\nu}^F = -\frac{i}{2} (\bar{\psi} \gamma_{(\mu} D_{\nu)} \psi - (D_{(\mu} \bar{\psi}) \gamma_{\nu)} \psi)$ symmetrised over the indices. This induces the following interaction terms with the scalar field

$$-\frac{i}{2} \int d^4x \sqrt{-g} \frac{\partial^\mu \phi \partial^\nu \phi}{M_\psi^4} (\bar{\psi} \gamma_{(\mu} D_{\nu)} \psi - (D_{(\mu} \bar{\psi}) \gamma_{\nu)} \psi) \quad (6)$$

where the mass scale M_ψ is the suppression scale for the fermion species ψ . A similar coupling was first considered in [12].

Let us focus on a typical situation where the metric is Minkowskian to a good approximation (this is the case on earth where Newton's potential is $\Phi_\oplus \sim 10^{-9}$) and the scalar field varies on scales much larger than the Compton wave length of the fermions. Moreover, let us assume that the scalar field is static. Then the interaction term reduces to

$$-\frac{i}{2} \int d^4x \sqrt{-g} d^i d^j (\bar{\psi} \gamma_i \partial_j \psi - \partial_i \bar{\psi} \gamma_j \psi) \quad (7)$$

where $d^i = \frac{\partial^i \phi}{M_\psi^2}$ is a slowly varying function of space only. Hence a static configuration of the scalar field yields a Lorentz violating interaction in the Fermion Lagrangian. The resulting Dirac equation becomes

$$i(-\gamma^0 \partial_0 + \gamma^i \partial_i + d^i d^j \gamma_i \partial_j) \psi = 0. \quad (8)$$

The dispersion relation is obtained by squaring the modified Dirac operator to obtain

$$p_0^2 = (c^2)^{ia} p_i p_a. \quad (9)$$

This becomes the dispersion relation in an anisotropic medium with a square velocity tensor

$$(c^2)^{ia} = (\delta^{ij} + d^i d^j)(\delta^{aj} + d^a d^j). \quad (10)$$

The eigenmodes of the velocity tensor are d^i and two vectors e_λ^i , $\lambda = 1, 2$ orthogonal to d^i . The eigenspeeds are $c_d = (1 + |d|^2)$ and twice $c_\lambda = 1$. Hence, fermions go faster than light in the direction of the gradient $\partial^i \phi$, i.e. along the scalar lines of force, with

$$\Delta c \equiv c_d - 1 = |d|^2. \quad (11)$$

An interesting application concerns fermions, typically neutrinos with no electromagnetic interactions, produced at the surface of a spherical body, traversing the sphere on a straight line parameterised by an angle θ varying between $-\theta_{\max}$ and θ_{\max} . We assume that d^i is radial.

Along this line, the fermion speed is $v(\theta) = 1 + |d|^2 \theta^2$ implying that fermions would be observed earlier compared to a propagation with the speed of light by

$$\frac{\Delta t}{t} = \frac{1}{3} |d|^2 \theta_{\max}^2. \quad (12)$$

A time difference of $\frac{\Delta t}{t} = 2.5 \cdot 10^{-5}$, as claimed by the OPERA experiment where $\theta_{\max} \sim 0.06$, would require $|d| \sim 0.15$. In the following we will use such a value as a template and unravel its physical consequences. If the OPERA claims turned out to be spurious, the mechanisms described here would still be valid and lower bounds on the scale M_ψ would ensue. These bounds would be easily extracted from the formalism discussed here.

When the scalar field is canonically normalised, the Klein Gordon equation is modified due to the coupling of the scalar field ϕ to matter:

$$\square \phi - \sum_i \frac{2}{M_i^4} D_\mu (\partial_\nu \phi T_i^{\mu\nu}) = -\beta \sum_i T_i + \frac{dV}{d\phi}, \quad (13)$$

where T_i is the trace of the energy momentum tensor $T_i^{\mu\nu}$ and the coupling of ϕ to matter is defined by $\beta \equiv m_{\text{Pl}} \frac{d \ln A}{d\phi}$. In static situations where matter is pressureless and space-time is assumed to be Minkowskian, the Klein-Gordon equation reduces to

$$\Delta \phi = -\beta T + \frac{dV}{d\phi} \quad (14)$$

corresponding to the case with no derivative coupling in the Lagrangian.

In a dense environment, the scalar field acquires a non-trivial profile due to the matter dependent source term. We will be interested in spherical situations corresponding to dense astronomical or astrophysical objects such as the earth or the sun. It turns out that the scalar-tensor theories of the type studied here would lead to large deviations from Newton's law in the solar system due the existence of a fifth force. This is prevented in chameleon-like theories[3] where the effective $V_{\text{eff}}(\phi) = V(\phi) + \rho_m A(\phi)$ acquires an environment dependent minimum where the mass is large enough to Yukawa screen the fifth force deep in the body, leaving only a thin shell of size ΔR over which the field varies significantly. Here we have defined ρ_m as the conserved matter density $T = -A(\phi) \rho_m$. In summary, for radial distances $r \leq R_s$ where R_s is the radius of the shell, the solution is constant $\phi = \phi_c$, $r \leq R_s$ where ϕ_c is the minimum of the effective potential inside the dense body and we assume $A(\phi) \approx 1$. In the thin shell, the field varies according to $\phi = \phi_\infty - \frac{\beta \rho_m R^2}{2m_{\text{Pl}}} + \frac{\beta \rho_m R^3}{3m_{\text{Pl}} r} + \frac{\beta \rho_m r^2}{6m_{\text{Pl}}}$. Outside the body, the fifth force is suppressed by a factor $\frac{3\Delta R}{R} \equiv \frac{|\phi_\infty - \phi_c|}{2\beta m_{\text{Pl}} \Phi_N}$ where Φ_N is the Newton potential generated by the body at its surface and $\Delta R = R - R_s$. A thin shell exists when

$3\Delta R/R \lesssim 1$. Solar system tests of gravity like the Lunar Ranging experiment require that the thin shell on earth should be such that $\beta \frac{\Delta R_\oplus}{R_\oplus} \lesssim 10^{-7}$ [3].

Inside a dense body, the scalar field has a non-vanishing gradient in a thin shell. More precisely we find that the gradient of the scalar field is radial $\partial^i \phi = \frac{d\phi}{dr} \frac{x^i}{r}$ where $\frac{d\phi}{dr} = -\frac{\beta \rho_m R_s^3}{3m_{\text{Pl}} r^2} + \frac{\beta \rho_m r}{3m_{\text{Pl}}}$, $R_s \leq r \leq R$, vanishes for $r \leq R_s$ and can be approximated by $\frac{d\phi}{dr} = \frac{\beta \rho_m}{2m_{\text{Pl}}} (r - R_s)$, $R_s \leq r \leq R$, which is maximal at the surface of the body and proportional to ΔR . As an example, let us consider fermions starting from the surface of the dense body and traversing the body at a small angle θ from the horizontal which starts at $-\theta_{\text{max}}$ and finishes at θ_{max} . Define by θ_{min} the angle when the fermions leave the thin shell. We have $\frac{d\phi}{dr} = \frac{\beta \rho_m R}{4m_{\text{Pl}}} (\theta^2 - \theta_{\text{min}}^2)$ for small angles and $\Delta\theta = \frac{\Delta R}{R\theta_{\text{max}}}$ with $\Delta\theta = \theta_{\text{max}} - \theta_{\text{min}}$. Along this trajectory the speed of the fermions is $v(\theta) = 1 + |d|^2 \theta^2$ corresponding to the difference of time of arrival between fermions and photons $\Delta t = \int_{\theta_{\text{min}}}^{\theta_{\text{max}}} (1 - \frac{d\theta}{1+|d|^2 \theta^2})$. Denoting by $t = R\theta_{\text{max}}$ the time photons take to go from $\theta_{\text{max}} \sim \theta_{\text{min}}$ to $\theta = 0$, we find that

$$\frac{\Delta t}{t} = \frac{R^2 \rho_m^2}{12\beta m_{\text{Pl}}^4 M_\psi^4} (\beta \frac{\Delta R}{R})^3. \quad (15)$$

As the thin shell is very small, this time difference can reproduce $\frac{\Delta t}{t} \sim 2.5 \cdot 10^{-5}$ with a low suppression scale $M_\psi \sim 2 \text{ eV}$.

Some models of modified gravity can be compatible with gravitational tests while describing scalar fields as being unscreened on earth. This is the case of the symmetron where the potential and the coupling functions are $V(\phi) = -\frac{\mu^2}{2} \phi^2 + \frac{\lambda}{4} \phi^4$, $A(\phi) = 1 + \frac{\phi^2}{2M_G^2}$. For a low energy density, the field is stabilised at a value $\phi_\star = \frac{\mu}{\sqrt{\lambda}}$ whilst for a large energy density $\rho \geq \rho_\star = \mu^2 M_G^2$, the minimum of the effective potential is at the origin. The solution inside an unscreened body reads $\phi(r) = A \frac{R}{r} \sinh \frac{\sqrt{\rho_m}}{M_G} r$ where $A \sim \frac{M_G^3}{m_{\text{Pl}}^2} \frac{1}{\sqrt{6\Phi_N}}$ with $M_G \leq 10^{-3} m_{\text{Pl}}$ to comply with solar system tests. In particular we have $\frac{d\phi}{dr}|_{r=R} = \mathcal{O}(\frac{A}{R})$. We will focus on the upper value of M_G which corresponds to $\phi_\star \sim 10^{-6} m_{\text{Pl}}$. A contribution of order $|d| \sim 0.15$ could be explained by these models provided $M_\psi^2 \sim 10A/R$, or equivalently $M_\psi \sim 7 \cdot 10^{-4} \text{ GeV}$.

Galileon models [11] depend on four unknown parameters and involve three non-canonical contributions to the kinetic terms. We consider the case with $c_{4,5} = 0$ for simplicity. Inside the Vainshtein radius expressed as $R_\star = (4c_3 \Phi_N \frac{R}{m_{\text{Pl}}^2})^{1/3}$ the Galileon profile due to a spherical mass M reads $\frac{d\phi}{dr} = \frac{M}{2m_{\text{Pl}} r^2} (\frac{r}{R_\star})^{3/2}$ where we have normalised the galileon choosing $c_2 = m_{\text{Pl}}^2$. In this case, we find that the gradient of the scalar field is $\frac{d\phi}{dr}|_R = m_{\text{Pl}}^2 (\frac{\Phi_N}{4c_3})^{1/3}$. Taking $c_3 \sim 10^{120}$ [13, 14] to satisfy the Lunar Ranging tests, we find that $d \sim 0.15$ could be due to

a galileon coupled to matter provided $M_\psi \sim 1.3 \text{ MeV}$.

Of course, we need to check that the neutrinos emerging from the supernova SN1987A should not be delayed compared to photons. Let us model the trajectory of the neutrinos as radial from earth, on the verge of the milky way at a distance $R_E \sim 8 \text{ kpc}$ from the galactic centre, to a distance $R_{\text{SN}} \sim 60 \text{ kpc}$. This is not the exact trajectory although it will give us the order of magnitude of the deviation. Along this trajectory, the neutrino speed varies as $c_d = 1 + |d|^2$ where $d = M_\psi^{-2} \frac{d\phi}{dr}$. The time difference between neutrinos and photons can be easily evaluated in the chameleon, symmetron and Galileon cases. For chameleons and symmetrons, we find that

$$\frac{\Delta t}{t} = \frac{1}{3} \frac{m_{\text{Pl}}^2 R_E^2}{R_{\text{SN}} M_\psi^4 \alpha_G^2} (\frac{1}{R_E^3} - \frac{1}{R_{\text{SN}}^3}) \quad (16)$$

where $\Phi_N \sim 10^{-6}$, $\alpha_G = \frac{R_G}{3\Delta R_G} \sim 10$ for chameleons and $\alpha_G = 6 \frac{m_{\text{Pl}}^2}{M_G^2} \Phi_N \sim 10$ for symmetrons. Typically we find that $\frac{\Delta t}{t} \sim 4 \cdot 10^{-19}$ in both cases. Similarly in the Galileon case we have

$$\frac{\Delta t}{t} = 4\Phi_N^2 \frac{(m_{\text{Pl}} R_E)^2}{(M R_\star)^4} \frac{R_\star}{R_{\text{SN}}} \ln \frac{R_{\text{SN}}}{R_E} \quad (17)$$

where the Vainshtein radius of the milky way is $R_\star \sim 160 \text{ Mpc}$. Hence we find that $\frac{\Delta t}{t} \sim 6 \cdot 10^{-33}$. Therefore, chameleons, symmetrons and Galileons could provide an explanation for the non-observation of a deviation from the speed of light for neutrinos emerging from SN1987A.

In the chameleon, symmetron and Galileon cases, we have found that a compatibility with $\frac{\Delta t}{t} \sim 2.5 \cdot 10^{-5}$ could be reached for low values of M_ψ . A low suppression scale could imply large effects in particle physics experiments such as the ones at LEP, i.e. increasing the width of the Z boson or modifying the electroweak precision tests, if the gauge kinetic terms also involve a low scale $M_F \sim M_\psi$. Although a detailed study is beyond the present work, a simplified analysis can be carried out. One can evaluate the order of magnitude of such effects by reducing the bilinear coupling of the scalar field to W and Z bosons, to a linear coupling. First of all notice that the energy momentum of gauge field involve $T_{\mu\nu}^F \supset \frac{1}{4} g_{\mu\nu} F^2$ leading to the effective coupling between the scalar field and the gauge bosons $\mathcal{L}_I = -\frac{1}{4M_F^4} (\partial\phi)^2 F^2$. In the vacuum where particle physics experiments take place, in the chameleon and symmetron cases, and expanding $\phi = \phi_{\text{vac}} + \delta\phi$, this leads to the operator, after one integration by parts, $\mathcal{L}_I \supset \frac{\phi_{\text{vac}}}{4M_F^4} \partial^2 \delta\phi F^2$. The gauge boson vacuum polarisation diagrams receive contributions from scalar loops. The effect of these loop is to induce potentially divergent contributions to the precision parameters S, T , etc ... Fortunately, at high momentum the electroweak symmetry breaking is irrelevant, implying a cancelation of the UV divergences. This is the essence of the screening theorem for scalars. As a result, only momenta up to the breaking

scale M_Z are relevant. This implies that we can replace the previous operator by $\mathcal{L}_I \sim \frac{\phi_{\text{vac}} M_Z^2}{4M_F^4} \delta\phi F^2$. The effect of such a vertex was studied in [15] where the suppressions scale $\hat{M}_F = \frac{4M_F^4}{\phi_{\text{vac}} M_Z^2}$ was constrained to be $\hat{M}_F \geq 1\text{TeV}$. Here we find that $\hat{M}_F \sim 10^{-35}\text{GeV}$ for $M_F \sim M_\psi$ in the symmetron case where $\phi_{\text{vac}} = \phi_\star$, a situation which is strongly excluded. The precision test bound is satisfied provided $M_F \geq 14\text{TeV}$. For chameleons[3], $\phi_{\text{vac}} \leq 10^{-28} m_{\text{Pl}}$, implying that $M_F \geq 0.15\text{GeV}$. In the Galileon case, the Lorentz invariant breaking background leads to the operator $\mathcal{L}_I = -\frac{dM_\psi^2}{4M_F^4} \partial_r \delta\phi F^2$ where $d \sim 0.15$, which would lead to the same effect in precision tests as $\mathcal{L}_I \sim -\frac{dM_Z M_\psi^2}{4M_F^4} \delta\phi F^2$ corresponding to a scale $\hat{M}_F = \frac{4M_F^4}{dM_Z M_\psi^2}$ which is $\hat{M}_F \sim 10^{-6}\text{GeV}$ when $M_F \sim M_\psi$. The precision test bound is satisfied provided $M_F \geq 100\text{MeV}$.

If the coupling to $g_{\mu\nu}^i$ respects gauge invariance and if flavour effects are taken into account [16, 17], all neutrinos of the standard model should couple with the same scale M_ψ . This would also entail that electrons and muons would have the same speed as the neutrinos at the classical level inside the earth. In the atmosphere there is a drastic difference between models with a thin shell effect like chameleons for which $\phi \sim \phi_{\text{atm}}$ sits at the minimum of the effective potential [3] and induces no change in the speed of fermions at all and models with no thin shell like symmetrons and Galileons where d is nearly constant in the atmosphere. The latter would strongly violate the bounds on the deviation from the speed of light for electrons as they are in the 10^{-15} range[17]. One possibility for these models would be to have an environmentally dependent violation of gauge invariance with $M_\nu \neq M_f$ for $f = e, \mu, \tau$. A large value of $M_f \geq 10^4 M_\nu$ would be enough to satisfy the experimental bounds. In vacuum where the scalar field has no gradient, gauge invariance would be retrieved. Unfortunately, the resulting large difference between the neutrino and the electron speeds inside the earth would lead to too much Cerenkov radiation and is therefore excluded[16].

The only viable models are the chameleon ones where d vanishes in the atmosphere and in the vacuum pipes of particle experiments, implying no deviation between the fermion speed and the speed of light there. Moreover, the fact that neutrinos are obtained from pion decay in vacuum where the neutrino speed is the speed of light implies that no bound on the resulting neutrino energy applies [18]. Similarly, in matter such as inside the earth, as long as all fermions couple to the same metric with the same M_ψ , no e^+e^- Cerenkov radiation is induced. For chameleon models, Cerenkov photon radiation $\nu \rightarrow \nu + \gamma$ happens in the thin shell only and leads to an energy loss

$$\frac{\delta E}{E} \approx -\frac{k}{10752\pi^4} \frac{\alpha G_F^2 E^5 \rho_m^6 R_\oplus^7}{\beta_{\text{max}} m_{\text{Pl}}^6 M_\psi^{12}} \left(\frac{\beta \Delta R_\oplus}{R_\oplus}\right)^7 \lesssim 10^{-8} \quad (18)$$

where $k = 25/448$ and we have taken neutrinos of energy $E = 20\text{GeV}$ and a suppression scale of $M_\psi \sim 2\text{eV}$. This tiny effect is due to the very small size of the thin shell on earth which results from the constraints coming from the Lunar Ranging experiment.

So far we have taken $\frac{\Delta t}{t} \sim 2.5 \cdot 10^{-5}$. If the OPERA claims were not confirmed and $\frac{\Delta t}{t}$ happened to be smaller but still greater than the bound on the deviation of the speed of charged leptons from the speed of light, i.e. $\frac{\Delta t}{t} \gtrsim 10^{-15}$, modified gravity models with no thin shell such as Galileons and symmetrons would not provide an explanation for such a phenomenon. Indeed, the deviation from the speed of light for neutrinos and charged leptons would be the same both inside the earth and in the atmosphere, implying a direct violation of the bound on the deviation of the speed of charged leptons from the speed of light. On the other hand, chameleons would still be able to reproduce a value of $\frac{\Delta t}{t} \gtrsim 10^{-15}$ with no violation of the bound on the speed of charged leptons deduced from experiments in particle accelerator vacua and the earth's atmosphere. Indeed, the gradient of the chameleon field vanishes inside these environments. As a result, this would simply imply that M_ψ would be larger than 2eV . Eventually, if experiments concluded that no significant deviation from the speed of light for neutrinos and charged leptons could be detected, this could simply mean that M_ψ is large enough to lead to no experimentally detectable effects. Of course, this would not rule out the possible existence of a coupling between hypothetical scalar fields modifying gravity and standard model particles.

If scalar fields modifying the laws of gravity exist, then the profile of these scalar fields in the presence of matter breaks Lorentz invariance. This Lorentz invariance violation could then be transmitted to the neutrino sector of the standard model as we have seen in (5). In particular, the putative difference of the neutrino speed from the speed of light in matter could be induced in simple modified gravity models such as the chameleon ones. In these models, as the variation of the neutrino speed follows the scalar lines of force, the deviation from the speed of light would be maximal for neutrinos emitted radially towards the centre of the earth and detected in a laboratory symmetrical from the emission point. In this case, the time difference would be

$$\frac{\Delta t}{t} = \frac{1}{R} \int_0^R dr d^2(r). \quad (19)$$

which depends on the entire profile of the scalar field inside the earth. For chameleons, the gradient of the scalar field is only non-zero in a thin shell implying that $\frac{\Delta t}{t}$ is identical to (15) for all neutrino energies, therefore distinguishing between chameleons and models with an energy dependent neutrino speed. In all cases, the comparison between the results of a neutrino-through-earth experiment and another one like OPERA where neutrinos re-

main close to the surface of the earth would give us crucial information, in particular on the gradient of scalar fields coupled to matter inside dense bodies. This would test the properties of elementary particles in dense environments, a domain which could lead to surprises due the hypothetical presence of scalar field gradients modifying the laws of gravity. This is a largely unexplored connection between gravitational physics and particle physics where the coupling of matter with scalar fields could be detected.

In conclusion, we have shown that modified gravity models with screening properties can induce Lorentz violation effects in the fermionic sector of the standard model. Such violations are induced by the profile of a scalar field coupled to matter in the presence of pressureless over densities. In particular, fermions have a space-dependent speed along the scalar lines of force. This could be revealed by sending neutrinos through the earth at different energies and measuring the time difference from their expected journey at the speed of light. More generally, exploring the possible coupling of standard model particles with scalar field modifying the laws of gravity is an exciting prospect, still in its infancy, and worth pursuing.

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